VTOL Control Power Requirements Reappraised

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One of the crucial items in the design of operational VTOL aircraft is the specification of realistic control power requirements for low-speed flight conditions. Various attempts to develop empirical design requirements have yielded inconsistent results that have often been subject to individual interpretation. This paper examines the problem from a fundamental, analytical viewpoint, viz., the computation of control power as the output of a randomly disturbed mechanical system. Use of the proposed method also permits the rational consideration of closely related handling quality aspects. Contrary to data available in the VTOL literature, this paper shows that control system sensitivity and power are not strongly related, although both can affect the vehicle handling qualities. The method probably exhibits its greatest utility as a preliminary design tool, where it can serve to evaluate competing system designs quantitatively, even with estimated system characteristics. Numerical examples are presented to illustrate the application of the method. The main disadvantages of the method are the requirements for detailed knowledge of the maneuver requirements, the atmospheric turbulence, and the airframe stability derivatives. An intimate knowledge of analytical handling qualities is also a prerequisite.

Nomenclature

CP = control power, rad/sec^2
CP_{stab} = stabilization control power, rad/sec^2
db = decibels = $20 \log_{10} |\ \ |$
$G_a(s)$ = actuator transfer function, rad/rad
$G_f(s)$ = feedback system transfer function, rad/rad
h = altitude, ft
h_0 = terrain roughness factor, ft
j = $(-1)^{1/2}$
K_a = control system gain, rad/rad
K_p = static gain term in pilot model, rad/rad
L = scale factor of turbulence, ft
$L_{\delta a}$ = rolling acceleration due to effective aileron input, rad/sec^2
$L_{\delta a}'$ = $[L_{\delta a} + (I_{xz}/I_x)N_\delta]_a/[1 - (I_{xz}^2/I_xI_z)]$
$P[a \geq b]$ = probability that $a \geq b$
s = complex variable, $\sigma + j\omega$, sec^{-1}
T = sample length, sec
t = time, sec
T_I = pilot-generated lag time constant, sec
T_L = pilot lead time constant, sec
T_N = pilot neuromuscular lag time constant, sec
U_0 = trim air speed, fps
V = mean wind speed, fps
v_g = random side gust velocity, fps
Y_p = quasi-linear pilot transfer function, rad/rad
Y_{CL} = closed-loop transfer function, $Y_{OL}(s)/[1 + Y_{OL}(s)]$
Y_{OL} = open-loop transfer function, always used in a local context
δ_a = effective aileron deflection measured from the trim position, rad
δ_a^* = complex conjugate of $\delta_a(s)$, rad
δ_c = cockpit control deflection measured from the trim position, rad
δ_L = arbitrary value of δ_a, rad
λ = nondimensional frequency
σ = real part of the complex variable s, (sec)$^{-1}$
σ_{vg} = root-mean-square gust velocity, fps
$\sigma_{\delta a}$ = root-mean-square aileron deflection, rad or deg
$\sigma_{\delta c}$ = root-mean-square cockpit control deflection, rad or deg
σ_ϕ = root-mean-square roll angle, rad or deg
τ = pilot reaction time delay, sec
ϕ = roll angle, rad or deg
$\Phi_{vg}(s)$ = roll/side gust velocity transfer function
$\Phi_{\delta a}(s)$ = roll/aileron deflection transfer function
$\Phi_{vgvg}(\omega)$ = power spectral density of atmospheric turbulence, ft^2/sec
$\Phi_{\delta a \delta a}(\omega)$ = power spectral density of δ_a, sec
ω = damped frequency, rad/sec
∞ = arbitrarily large
$|\ \ |$ = magnitude

Introduction

VALID control power requirements for vertical takeoff and landing (VTOL) aircraft are absolutely essential to the aircraft designer and must be available in the early design phase of any new operational aircraft for which the performance requirements are stringent. This includes most conceivable future civilian and military aircraft. The performance penalties exacted for overly conservative allowances for this quantity are well known and can be severe. The penalty to be paid for an unconservative allowance can be even more severe.

If one is to examine the literature, he will find at least 14 generally available documents that discuss the question of specification of minimum control power requirements for VTOL aircraft or that present data to aid in the establishment of such specifications. Other less-available documents also exist (usually summarizing in-house research efforts done by various airframe manufacturers, both domestic and foreign) which discuss the same topic. It would appear, then, that any new paper on this subject, such as the present one, must be required to offer some justification for its existence. First let us examine what today's literature *really* contains.

All available reports on this subject are empirically based. Test-bed aircraft have been flown and simulator experiments performed for the purpose of defining the minimum necessary control power levels with which any operational VTOL aircraft must be supplied in order to permit control of the vehicle's attitude and flight path. All references claim to provide a rational basis from which suitable, quantitative requirements can be derived.

However, little commonality exists among these data. Wide discrepancies are the rule rather than the exception. There is even confusion among researchers concerning use of such terms as "control power" and "control sensitivity." In at least one instance, this author has seen the same data presented as "control power" in one source and as "control

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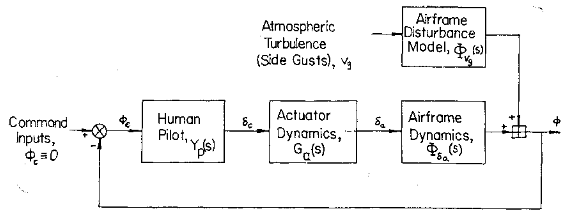

Fig. 1 Typical single-axis roll control system, unaugmented airframe.

sensitivity" in another. This sort of ambiguity is beautifully illustrated in Ref. 1.

As a result of what might generously be called "confusion" regarding this critical point, the aircraft designer has been placed in the position of having no rational basis for estimating required system control power levels for new aircraft—which must be done in the early design stages because of possible interaction with the propulsion system and, hence, performance requirements.

A critical assessment of existing literature suggests, then, that there is indeed a requirement for another examination of the control power problem, but from a new vantage point. It is in this light that the present paper is submitted. The author's reasons for choice of its title should be self-evident.

Analysis

Aircraft stability and maneuverability are distinct (in fact, conflicting) properties. Control inputs are required to stabilize the aircraft attitude, for example, in some disturbance field, whereas additional control inputs are necessary to maneuver the vehicle. If it is recognized that the stabilization and maneuvering functions for any flight control system are independent, then we are free to examine the two modes separately. Specifically, this implies that the control power requirements for a nonmaneuvering VTOL aircraft (or any aircraft for that matter) can be deduced independently of the maneuvering control power requirements. This is the basis for the analysis presented in this paper.

The total control input required for low-speed, low-altitude flight conditions will be considered specifically in this paper. For brevity, the proposed analytical method will be introduced through the medium of example problems. The applicability of the method to other aircraft or other flight conditions will usually be obvious and will not be discussed per se.

The Doak VZ-4 tilt-duct VTOL aircraft will serve as the example aerodynamic configuration for all numerical examples. The required dynamic and aerodynamic data for this airplane will be used as compiled in Ref. 2. These data represent the culmination of intensive analysis and yield response characteristics that agree reasonably well with flight tests. It will be assumed that the VZ-4 configuration represents a preliminary design for an actual operational vehicle and that the control system design has not been finalized. In other words, the example analysis could serve as a prototype for an actual, practical design analysis.

In addition, the same flight condition will be employed for all numerical examples, that is, hovering over a fixed ground point in turbulent air with a 35-knot headwind, and 1) flight path angle = 0; 2) aircraft velocity relative to air mass U_0 = 35 knots = 59 fps; 3) altitude h = 100 ft; 4) terrain roughness factor h_0 = 15 ft; and 5) mean wind speed V = 35 knots.

The reader will note that all control inputs are measured from their trim position. The control power required to establish or maintain a trim point will not be considered since this is dependent upon the particular configuration. The term "total control power," as used in this paper, refers only to the control power required to stabilize and maneuver the trimmed airframe. An actual design analysis must also allow for the trim control power, as well as for those components discussed in this paper.

Stabilization Control Power

The six-degree-of-freedom, rigid-body equations of motion for any aircraft, including VTOL, are nonlinear and involve coupling between all six degrees of freedom. Can these equations be linearized and uncoupled on any rational basis for the VTOL aircraft in low-speed flight?

Consider an airplane in initially steady, undisturbed, wings-level flight. Using a stability axis system and assuming a fuselage-level configuration, the inertial and body forces appearing in the side-force equation, the rolling-moment equation, and the yawing-moment equation may be written, respectively, as

$$\Sigma F'_y = m(\dot{v} + U_0 r + ru - wp - g \sin\phi \cos\theta)$$

$$\Sigma L = \dot{p} I_x - \dot{r} I_{xz} + qr(I_z - I_y) - pqI_{xz}$$

$$\Sigma N = \dot{r} I_z - \dot{p} I_{xz} + pq(I_y - I_x) + qrI_{xz}$$

For VTOL aircraft, as in the conventional case, we can legitimately assume that products of rotational perturbation quantities are negligible and that all angular rotations are very small. When the vehicle's steady-state velocity approaches zero, it does not appear possible to rigorously justify neglecting the product terms involving both rotational and linear perturbation quantities. Instead, we must resort to physical arguments.

In the case of the conventional helicopter and possibly for most operational VTOL as well, the amount of coupling involved is small. In fact, any such coupling *must* be small if the vehicle's handling qualities are not to suffer. Any residual coupling would be removed either by the pilot or by the control system. Then it will be assumed that ru and wp are negligible compared with $\dot{v}$ and $g\phi$. This doesn't appear to be unreasonable.

With these assumptions, the expressions for the body and inertial forces become

$$\Sigma Fy' = m(\dot{v} + U_0 r - g\phi)$$

$$\Sigma L = \dot{p} I_x - \dot{r} I_{xz}$$

$$\Sigma N = \dot{r} I_z - \dot{p} I_{xz}$$

which, when equated to the aerodynamic and control forces, will be recognized as the conventional, linear, small-perturbation equations of motion in which the lateral motions are uncoupled from the longitudinal ones.

No attempt will be made to consider the transition maneuver, since this involves time-varying equations. Only perturbations about a steady-state flight condition at very low forward speeds will be considered. This steady state could correspond to a "frozen point" on an actual transition profile.

The example problems will consider only the one reference flight condition. Stabilization control power estimates for all other conditions can be obtained in a similar manner.

Only the simple roll control systems of Figs. 1 and 2 will be considered. These are practical systems of great interest, yet reasonably simple, and should provide suitable models to demonstrate the analytical methods. As shown in the figures, only manual control systems will be considered.

It will be assumed that the atmospheric turbulence is random, isotropic, and has a Gaussian amplitude distribution. The simplified empirical gust spectral form given (incorrectly) on page 67 of Ref. 3 will be used (as corrected) to represent the random side gust spectra. That is, the gust power spectral density is given by

$$\Phi_{v_g v_g}(\omega) = (4\sigma_{v_g}{}^2 L/U_0)[1/(1 + \lambda^2)]$$

where $\lambda = L\omega/U_0$, and

$$\sigma_{vg}^2 = \frac{1}{2\pi}\int_0^\infty \Phi_{vgvg}(\omega)d\omega$$

is the mean-square side gust. Reference 3 quotes Panofsky as suggesting $\sigma_{vg} = 0.226V/[\log(h/h_0)]$ and $L = 0.93h$ for low-altitude turbulence. Then for the given flight condition, upon substituting the proper numerical values and performing some simple arithmetic, find

$$\Phi_{vgvg}(\omega) = |40.8/[(S/0.633) + 1]|^2_{s=j\omega} \quad \text{ft}^2/\text{sec} \qquad (1)$$

With the system disturbance described only in statistical form, the response of any system element can be discussed only in a probabilistic sense. For example, there exists a unique, finite probability that $\delta_a(t)$ will equal or exceed a specified level δ_L during some arbitrary time period.

So that there can be no ambiguity concerning the physical distinction between "control power" and "control system sensitivity," let us define the terms now. First, it will be assumed throughout the following discussion that the curve of rolling moment vs effective aileron deflection is *linear* for rolling moments equal to or less than the predicted value necessary for providing adequate stabilization control power.

Then *define* the "instantaneous control power" $CP(t)$ as the rolling moment produced by some effective aileron deflection $\delta_a(t)$ divided by an appropriate constant (in this example, the rolling moment of inertia). That is,

$$CP(t) \equiv (1/I_x)[\partial(\text{rolling moment})/\partial\delta_a] \times \delta_a(t) \equiv L_{\delta a}\delta_a(t) \text{ rad/sec}^2$$

This definition of control power is generally consistent with existing literature. The purpose of the present analysis is to estimate the minimum required value for $CP_{\max}$. It is the quantity that is defined later as CP_{stab}. Also, it is this quantity that is of interest to the aircraft designer, since it partially establishes the propulsion system constraints. It may also be helpful to visualize CP_{stab} as a measure of the energy required to stabilize the vehicle in the presence of disturbances.

For simplicity, and partly because it introduces little error in this application, it will be assumed that the control system dynamics may be represented as a static gain. That is,

$$G_a(s) = K_a \qquad (2)$$

Then the instantaneous rolling control power per unit of cockpit control deflection is given entirely by the product $K_aL_{\delta a}$. This quantity will be defined as the "control system sensitivity." This definition is also generally consistent with existing literature.

The sensitivity $K_aL_{\delta a}$ is a design parameter that can be adjusted at will by the flight control designer. In a manual system such as this one, the sensitivity is normally adjusted so as to optimize pilot opinion of the roll tracking system. The present analysis will establish that, despite the attempts of numerous investigators to establish a correlation between control power requirements and control system sensitivity, these quantities are not strongly related.

It should be observed that the control deflection δ_a is an artifact for the Doak VZ-4 in the low-speed configurations. The term δ_a does not exist as a distinct entity but, rather, as a combination of aileron deflection and deflection of inlet guide vanes in each duct. This does not affect the analysis, however, since the net control effect is entirely expressed by δ_a. In addition, it should be noted that δ_a is measured from the trim position for this flight condition.

Now, since the system is linear and the disturbance is random with a Gaussian amplitude distribution, then it is also true that $CP(t)$ is a random variable with Gaussian amplitude distribution. At this point, the analysis must be

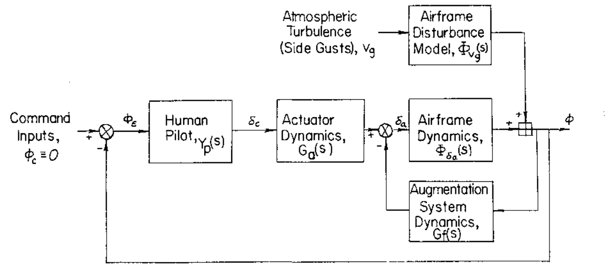

Fig. 2 Typical single-axis roll control system, augmented airframe.

compromised in order to satisfy physical reality, viz., the VTOL control system cannot be supplied with unlimited control power, nor can $\delta_a(t)$ or $\phi(t)$ increase without bounds in any physical system, as is implied by the choice of a Gaussian distribution. Accordingly, the stabilization control power will be defined as some value $CP_{\text{stab}} = L_{\delta a}\delta_L$ such that, if the system were completely linear, the probability of $CP(t)$ exceeding CP_{stab} at this flight condition would be arbitrarily small. The influence of control power limiting on the linear system analysis will be ignored, together with the statistical implications of finite time periods. These assumptions are made not out of necessity but, rather, to avoid complicating this rather simple analysis by introducing describing function techniques and exceedance criteria. A satisfactorily rigorous analysis could be performed with the methods of Refs. 4 and 5.

It should be emphasized that the installed control power saturates at CP_{stab} only if the control power required for stabilization (at the specified trim condition) exceeds that required for the most severe combination of stabilization and maneuvering. This latter requirement will be discussed in a subsequent section. If the total control power available exceeds CP_{stab}, however, there may be no requirement that the $CP(t)$ vs δ_c curve remain linear for $CP > CP_{\text{stab}}$.

For an automatic control system, it is probable that the choice of $CP_{\text{stab}} = 3L_{\delta a}\sigma_{\delta a}$ is reasonably conservative. This choice implies that $P[CP(t) \geq CP_{\text{stab}}] = 0.0027$ for any arbitrary time (e.g., Ref. 6). This value of $\delta_L = 3\sigma_{\delta a}$ is known popularly as the "three-sigma" value.

For a manually controlled system, additional complications arise. It is known[7] that, in tracking tasks similar to this one, the human operator's dynamic response to random inputs is predominately, but not completely, linear. When the analyst is chiefly concerned with system stability or tracking performance, this nonlinear operator response is of relatively little consequence, especially for higher-order, realistic vehicle dynamics. However, predictions of operator (pilot) output, in terms of $\sigma_{\delta a}$, which fail to include the nonlinear effect might be unconservative. Until more definitive information is available, it is suggested that the "four-sigma" value of δ_a will be a suitably conservative representation for the maximum control input for this stabilization task. Then for manual control it will be assumed that

$$CP_{\text{stab}} = 4L_{\delta a}\sigma_{\delta a} \qquad (3)$$

The quantity $\sigma_{\delta a}$ will be obtained by application of statistical design methods. (See Ref. 8 for a brief introduction.) It can be shown that

$$\sigma_{\delta a}^2 = \frac{1}{2\pi}\int_0^\infty \Phi_{\delta_a\delta_a}(\omega)d\omega \qquad (4)$$

where

$$\Phi_{\delta a\delta a}(\omega) = \lim_{T\to\infty}(1/T)[\delta_a^*(j\omega)\delta_a(j\omega)] \qquad (5)$$

is called the power spectral density of $\delta_a(t)$.

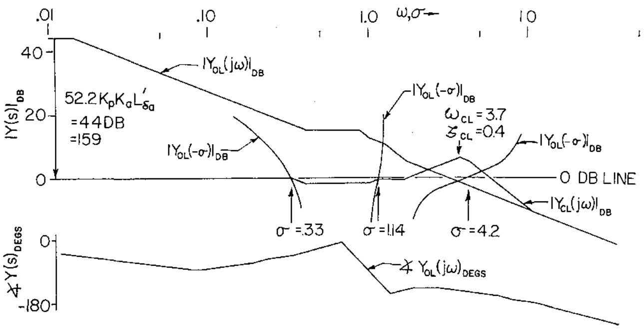

Fig. 3 Bode plots for the unaugmented pilot-vehicle system.

For the unaugmented airframe (Fig. 1), it is easy to see that

$$\delta_a(s) = -Y_p(s)K_a\phi(s) \tag{6}$$

and, using simple block-diagram algebra, it can be shown that

$$\phi(s) = [Y_{CL}(s)/Y_{OL}(s)]\Phi_{vg}(s)vg(s) \tag{7}$$

where

$$Y_{OL}(s) = Y_p(s)K_a\Phi_{\delta a}(s) \tag{8}$$

and

$$Y_{CL}(s) = Y_{OL}(s)/[1 + Y_{OL}(s)] \tag{9}$$

Hence, from Eqs. (5–9), obtain

$$\Phi_{\delta a\delta a}(\omega) = |Y_{CL}(s)[\Phi_{vg}(s)/\Phi_{\delta a}(s)]|^2_{s=j\omega} \lim_{T\to\infty}[v_g{}^*(j\omega)v_g(j\omega)]$$

But, by definition,

$$\Phi_{vgvg}(\omega) = \lim_{T\to\infty}(1/T)[v_g{}^*(j\omega)v_g(j\omega)]$$

so that

$$\Phi_{\delta a\delta a}(\omega) = |Y_{CL}(s)[\Phi_{vg}(s)/\Phi_{\delta a}(s)]|^2_{s=j\omega}\Phi_{vgvg}(\omega) \tag{10}$$

Substitution of Eqs. (1) and (10) into Eq. (4) yields

$$\sigma_{\delta a}{}^2 = \frac{1}{2\pi}\int_0^\infty \left| Y_{CL}(s)\frac{\Phi_{vg}(s)}{\Phi_{\delta a}(s)}\frac{40.8}{[s/0.633 + 1]}\right|^2_{s=j\omega} d\omega \text{ rad}^2 \tag{11}$$

for the manually controlled, unaugmented system.

In an exactly analogous manner, it can be shown that, for the manually controlled, augmented system (Fig. 2), Eq. (11) is also valid, provided that we use

$$Y_{CL}(s) = \frac{Y_{OL}(s)}{1 + Y_{OL}(s)} = \frac{[Y_p(s)K_a + G_f(s)]\Phi_{\delta a}(s)}{1 + [Y_p(s)K_a + G_f(s)]\Phi_{\delta a}(s)} \tag{12}$$

In addition it will be helpful to have expressions for the mean square error $\sigma_\phi{}^2$ for both of the example systems. It is easy to show that, for both systems,

$$\sigma_\phi{}^2 = \frac{1}{2\pi}\int_0^\infty \left|\frac{Y_{CL}(s)}{Y_{OL}(s)}\frac{40.8}{\left(\frac{s}{0.633} + 1\right)}\Phi_{vg}(s)\right|^2_{s=j\omega} d\omega \tag{13}$$

where Y_{OL} and Y_{CL} are given by Eqs. (8) and (9) for the unaugmented system and by Eq. (12) for the augmented case.

The computation of both CP_{stab} and $\sigma_\phi{}^2$ will require the airframe transfer functions, $\Phi_{\delta a}(s)$ and $\Phi_{vg}(s)$. These may be computed from the complete, linearized, lateral-directional equations of motion for aileron and gust inputs. The analysis may be further simplified by assuming that the entire effect of a random side gust is to introduce random perturbations of the sideslip angle. This assumption neglects effects due to gust distribution over the airframe and is not completely realistic, but it *is* simple. The airframe equations of motion are given in Eqs. (14):

$$\begin{aligned}
(s - Y_v)v(s) - (Y_ps + g)\Phi(s) + (U_0 - Y_r)r(s) &= Y_{\delta a}\delta_a(s) + Y_vv_g(s)\\
-L_v\, v(s) + (s^2 - L_ps)\Phi(s) - [(I_{xz}/I_x)s + L_r]r(s) &= L_{\delta a}\delta_a(s) + L_vv_g(s)\\
-N_v\, v(s) - [(I_{xz}/I_z)s^2 + N_ps]\phi(s) + (s - N_r)r(s) &= N_{\delta a}\delta_a(s) + N_vv_g(s)
\end{aligned} \tag{14}$$

The estimated aerodynamic stability derivatives for the Doak VZ-4 are contained in Ref. 2 and are repeated in Table 1 for convenience. Using these derivatives and eqs. (14), the required airframe transfer functions are found to be approximately

$$\Phi_{\delta a}(s) = \frac{52.2L_{\delta a}{}'\left(\frac{s}{0.407} + 1\right)\left(\frac{s}{1.712} + 1\right)}{\left(\frac{s}{0.01426}\right) + 1\left(\frac{s}{1.224} + 1\right) \times \left[\left(\frac{s}{0.868}\right)^2 + \frac{2(0.1635)}{0.868}s + 1\right]} \tag{15}$$

$$\Phi_{vg}(s) = \frac{-0.0357s\left(\frac{s}{0.0204} + 1\right)}{\left(\frac{s}{0.01426} + 1\right)\left(\frac{s}{1.224} + 1\right) \times \left[\left(\frac{s}{0.868}\right)^2 + \frac{2(0.1635)}{0.868}s + 1\right]} \tag{16}$$

The human pilot model $Y_p(s)$ will also be required. The selection of this model is not a straightforward process, and, usually, a unique pilot model cannot be achieved. Very briefly, it has been shown[7] that the pilot dynamic response for this and similar tracking tasks may be represented by the quasi-linear describing function,

$$Y_p(s) = \frac{K_p(T_Ls + 1)e^{-\tau s}}{(T_Is + 1)(T_Ns + 1)} \tag{17}$$

which, although not capable of reproducing the pilot response in a point-by-point sense, will represent the time-averaged pilot response provided that the model parameters are properly chosen.

Rather than attempt to discuss the detailed selection procedure (which has been done in the cited references and is not the point of this paper), it will be stated only that there are definite rules that govern the choice of the pilot model parameters. These rules are complicated by the somewhat artistic nature of the selection process and the practice required for their application. Basically, the selection of a suitable pilot model is contingent upon the specific task constraints, the airframe-control-display system dynamics, the known pilot

Table 1 Doak VZ-4 lateral derivatives (level flight, U_0 = 58.8 fps)[2]

$Y_v = -0.2895$	$L_v = -0.0224$	$N_v = 0.0081$	$I_{xz}/I_x = -0.1245$
$Y_p = 0$	$L_p = -0.455$	$N_p = 0.0605$	$I_{xz}/I_z = -0.07188$
$Y_r = 0$	$L_r = 1.75$	$N_r = -0.655$	I_x = 1990 slug-ft^2
$Y_{\delta a} = -24.9$	$L_{\delta a} = 0.5013$	$N_{\delta a} = 0.003$	I_z = 3450 slug-ft^2
$Y_{\delta r} = 1.85$	$L_{\delta r} = -0.141$	$N_{\delta r} = -0.78$	W = 3100 lb

dynamic capabilities, and the manner in which the pilot's subjective evaluation of the complete system is known to be influenced by various system dynamic characteristics.

The reader desiring to explore the intriguing area of human dynamic response further is referred to Ref. 7 for a comprehensive treatment of the subject, Ref. 9 for an excellent review of pilot-vehicle analysis techniques (including selection of the pilot model), Ref. 10 for an overview of the potential utility of the pilot model concept in the area of aircraft handling qualities, and Refs. 11–13 for interesting example applications.

Unaugmented airframe

Consider the unaugmented Doak VZ-4. Within the framework of the preceeding references, it can (but won't) be shown that a reasonable choice for the pilot model is

$$Y_p(s) = K_p\left(\frac{s}{1.0} + 1\right)e^{-0.2s} = \frac{K_p[(s/1.0) + 1][(s/-10) + 1]}{[(s/10) + 1]} \tag{18}$$

This model permits good closed-loop transient response, near-minimum tracking errors, and reasonably good pilot opinion ratings (POR), as reflected by the Cooper rating scale[14] or some suitable modification.

With this choice then, from Eqs. (8, 15, 18), obtain

$$Y_{OL}(s) = \frac{52.2K_pK_aL'_{\delta a}\left(\frac{s}{0.407} + 1\right)\left(\frac{s}{1.0} + 1\right) \times \left(\frac{s}{1.712} + 1\right)\left(\frac{s}{-10} + 1\right)}{\left(\frac{s}{0.01426} + 1\right)\left(\frac{s}{1.244} + 1\right)\left[\left(\frac{s}{0.868}\right)^2 + \frac{2(0.1635)}{0.868}s + 1\right]\left(\frac{s}{10} + 1\right)} \tag{19}$$

Having obtained $Y_{OL}(s)$, we may now compute $Y_{CL}(s)$ from Eq. (9). The foremost difficulty involved is the choice of open-loop gain, 52.2 $K_pK_aL_{\delta a}'$, such that the closed-loop tracking performance and transient response are acceptable. This, however, constitutes the classic problem of servomechanisms analysis, and any suitable technique may be used for its solution. The approximate servo analysis methods of Ref. 15 are especially recommended, since they involve only pencil and paper techniques, thereby giving the analyst an intimate appreciation for the more important physical aspects of the problem which may permit significant simplifications.

Bode plots for $Y_{OL}(j\omega)$ and $Y_{OL}(-\sigma)$ are shown in Fig. 3. Observe that here σ is the real part of the complex variable $s = \sigma + j\omega$ and is not to be confused with the rms values used elsewhere in this paper. The corresponding root locus plot is shown in Fig. 4. The choice of $52.2\,K_pK_aL_{\delta a}' = 159$ probably yields about the best combination of closed-loop transient response and tracking performance for the given open-loop dynamics. Using this value of open-loop gain and the "decomposition" method of Ref. 15 (a graphical method for obtaining Y_{CL} given Y_{OL}), it is found that

$$Y_{CL}(s) \doteq \frac{\left(\frac{s}{0.407} + 1\right)\left(\frac{s}{1.0} + 1\right)\left(\frac{s}{1.712} + 1\right)\left(\frac{s}{-10} + 1\right)}{\left(\frac{s}{0.33} + 1\right)\left(\frac{s}{1.14} + 1\right) \times \left[\left(\frac{s}{3.7}\right)^2 + \frac{2(0.4)}{3.7}s + 1\right]\left(\frac{s}{4.2} + 1\right)} \tag{20}$$

The tracking performance may be evaluated by computing σ_ϕ. Substitution of Eqs. (16, 19, and 20) into Eq. (13) yields

$$\sigma_\phi{}^2 \doteq \frac{0.0000838}{2\pi}\int_0^\infty \left|\frac{s\left(\frac{s}{0.0204} + 1\right)\left(\frac{s}{10} + 1\right)}{\left(\frac{s}{0.33} + 1\right)\left(\frac{s}{0.633} + 1\right)\left(\frac{s}{1.14} + 1\right)\left[\left(\frac{s}{3.7}\right)^2 + \frac{2(0.4)}{3.7}s + 1\right]\left(\frac{s}{4.2} + 1\right)}\right|^2_{s=j\omega} d\omega \tag{21}$$

Equation (21) may be integrated in any of several ways (see Ref. 8 for examples). By graphical integration, it is found that $\sigma_\phi{}^2 = 0.001305$ rad^2 or $\sigma_\phi = 2.07°$. This value is indicative of good manual tracking performance.

Substitution of Eqs. (15, 16, and 20) into Eq. (11) yields

$$\sigma_{\delta a}{}^2 \doteq \frac{0.000777}{2\pi(L_{\delta a}')^2}\int_0^\infty \left|\frac{s\left(\frac{s}{0.0204} + 1\right)\left(\frac{s}{-10} + 1\right)}{\left(\frac{s}{0.33} + 1\right)\left(\frac{s}{0.633} + 1\right)\left[\left(\frac{s}{3.7}\right)^2 + \frac{2(0.4)}{3.7}s + 1\right]\left(\frac{s}{4.2} + 1\right)}\right|^2_{s=j\omega} d\omega \tag{22}$$

Again, by graphical integration, obtain $\sigma_{\delta a}{}^2 = 0.0614/L_{\delta a}'^2$ or $\sigma_{\delta a} = 0.248/L_{\delta a}'$ rad. For the VZ-4 at this flight condition, the control and inertial cross-coupling terms are small so that $L_{\delta a} = L_{\delta a}'$ is a good approximation. Then substitution of the computed value of $\sigma_{\delta a}$ into Eq. (3) gives

$$CP_{\text{stab}} = 0.99 \text{ rad/sec}^2 \tag{23}$$

for the manually controlled, unaugmented VZ-4.

A very important observation can be made at this point: $\sigma_{\delta a} = K_a\sigma_{\delta c}$. Then $\sigma_{\delta c} = \sigma_{\delta a}/K_a \doteq 0.248/K_aL_{\delta a}$. Now, since the system is linear, the expression for the required power, Eq. (3), is equivalent to $CP_{\text{stab}} = 4K_aL_{\delta a}\sigma_{\delta c}$. But, since the control system sensitivity $K_aL_{\delta a}$ occurs as a factor in the denominator of $\sigma_{\delta c}$, then the computation of CP_{stab} is virtually *independent* of system sensitivity $K_aL_{\delta a}$, as stated previously.

The reader should be careful to differentiate between the total open-loop gain (52.2 $K_pK_aL_{\delta a}'$ in this problem), which does affect the computation of $\sigma_{\delta a}$ and therefore CP_{stab}, and

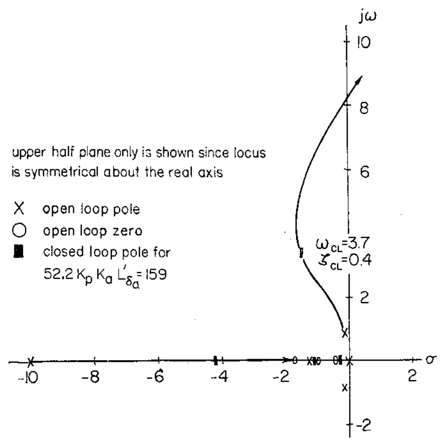

Fig. 4 Root locus plot for the unaugmented pilot-vehicle system.

the control sensitivity, $K_a L_{\delta a}$, which is merely a part of the total system gain. Although control sensitivity is not directly related to control power, handling quality considerations require that it be optimized with respect to pilot opinion rating in any actual system design.

In the opinion of this author, much of the past confusion concerning control power and sensitivity has materialized as a result of failure to understand that control sensitivity is an important handling quality parameter that is capable of influencing POR *independently* of control power. On the other hand, control power probably has a significant influence on POR only when it becomes "too small."

The VZ-4 flight test reports[16,17] state that the lateral control power was "poor." (The data of Ref. 17 imply that the lateral control power was "poor.") The data of Ref. 17 imply that the actual installed roll control power at this flight condition was only about 0.21 rad/sec^2 (other references specify slightly larger values), which, when compared with the predicted requirement for stabilization only, i.e., $CP_{\text{stab}} = 0.99$ rad/sec^2, appears woefully inadequate. Unfortunately, we cannot fully correlate the predicted control power with the flight test results, since neither Ref. 16 nor 17 permits the identification of poor ratings due to 1) insufficient control power for attitude control; 2) insufficient control power for rolling maneuvers; 3) basic handling quality considerations other than 1 or 2 such as discussed in Ref. 10; or 4) some combination of the preceding factors.

Augmented airframe

A simple, effective form of stability augmentation for the Doak VZ-4 would be to add a simple feedback of roll rate about the basic airframe. That is, in Fig. 2, we would choose

$$G_f(s) = K_f s \tag{24}$$

Again, the example will consider only the frozen point solution, i.e., K_f = const. In practice, it might actually be necessary that $G_f(s)$ be some form of adaptive controller, but such considerations are beyond the scope of this paper. Also, $G_f(s)$ would include washout circuitry in an actual installation, but that is ignored here.

Space does not permit presentation of the complete analysis for this case; however, it is reasonably straightforward. The data of Ref. 10 prove very useful in selecting the system gains in order to optimize the resulting pilot opinion. This simplifies the problem somewhat.

It can be shown that, if $K_f L_{\delta a}' = 16.9$ and $K_p K_a / K_f = 3.41$, then the handling qualities will be near optimum. Then, using Eqs. (12) and (11), compute $\sigma_{\delta a} = 0.195/L_{\delta a}'$. But, since $L_{\delta a} \doteq L_{\delta a}'$, the required stabilization control power [Eq. (3)] becomes

$$CP_{\text{stab}} = 0.78 \text{ rad/sec}^2 \tag{25}$$

From Eq. (13), it can be shown that $\sigma_\phi = 0.174°$.

Thus, we see that the effect of rate feedback has, in this example, 1) improved tracking accuracy by an order of magnitude; 2) decreased required stabilization control power by about 25%; and 3) probably effected significant improvement in roll-handling qualities. However, it should be recognized that these results may not be true in the universal sense.

It is pertinent to remark at this point that rate feedback has been employed, together with "control power," as a parameter in most of the VTOL handling qualities studies presented in the literature. The typical presentation of experimental results shows a three-parameter plot (with no data points!) of pilot opinion, "control power," and rate damping. This is nonsense!

Since Kauffman's[20] experiments with a variable stability F6F-3 airplane in 1948, it has been widely recognized and substantially reported in the literature that flying qualities (as reflected by pilot opinion) are a function of basic static and dynamic aircraft characteristics. Since rate feedback merely serves to modify the basic airframe dynamics, then it should be no mystery that such augmentation affects pilot opinion, regardless of any consideration of control power.

However, we have no assurance that all VTOL aircraft will use only rate feedback augmentation or, even if they do, that the amount of rate damping will be the same for all cases. This is true simply because all aircraft have different dynamic behavior and different operational requirements. Then it makes no sense to attempt to relate rate damping to control power or pilot opinion in a generally valid manner.

Maneuvering and Asymetric Conditions

The control power required to perform a prescribed maneuver can always be determined, in principle, by solving the aircraft equations of motion, given the aircraft motion, to obtain the necessary control inputs. Such analyses, termed inverse problems, are discussed by Etkin in Chap. 11 of Ref. 18.

Unfortunately, detailed knowledge of the aircraft motion involved in performing a maneuver is seldom known, although realistic estimates may often be made, e.g. Etkin's suggested roll angle response during a roll and stop maneuver.[18] Furthermore, we have no assurance that such "synthetic" maneuvers can, or will, be duplicated in normal operation.

An even more basic problem involves the specification and cataloging of all of the detailed maneuvers which a given aircraft may be required to perform during its operational lifetime. Possibly this cannot be realistically accomplished for any future system from a priori considerations alone. Classically, knowledge of system operational requirements often permits reasonable, albeit intuitive, estimates of those maneuvers (including failure modes) which weigh most heavily on the control system design. Experience will establish whether such a crutch will continue to serve the VTOL control system designer as well.

For those aircraft that are not required to perform very low-speed maneuvers, it is possible that the stabilization control power estimate will be approximately equal to the total required control power. If the aircraft is required to perform low-speed maneuvers or to satisfy severe asymmetry conditions (such as maintaining constant bank angle in a 30-knot sidewind), then it may happen that the maneuvering control power is greater than that required for stabilization.

Total Control Power

Given estimates of the control power required for both stabilization and maneuvering at one flight condition, it is necessary to estimate the total control power required by the system. The simple arithmetic sum of the two component control powers probably yields an overly conservative estimate of the total control power required, since this implies that both component maximums occur simultaneously, i.e., the system would be designed for the worst, perhaps highly improbable, condition.

The other extreme would be to choose the total control power to equal the larger of the two component control powers. If the required maneuvering control power is greater than CP_{stab}, this may be unconservative, since it implies that no stabilization control power is required during the design maneuver.

As an alternative to these extremes, it is suggested that the installed, maximum control power be chosen to equal either the maneuvering control power plus the one-sigma value of the stabilization control power or the four-sigma stabilization control power alone, whichever is larger. The author confesses that this is an opinionated alternative, based more on artistry than fact. This topic might contain the germ for additional fruitful research.

Comments

One of the most fundamental handling quality parameters is control system sensitivity. It was shown that control power and sensitivity are not strongly related parameters, although both can, under the proper conditions, strongly influence pilot opinion ratings. Considering the potential impact of this conclusion, additional comments are in order.

Sensitivity is primarily a closed-loop parameter that can have significant effects on POR for manual tracking tasks, but possibly ceases to be a dominant factor for open-loop flight modes. Control power is basically an open-loop parameter that must only be of sufficient size to permit performance of the critical tasks (either tracking or maneuvering); that is, a simulation in which sensitivity remains constant would probably show little effect of control power on POR for values of control power greater than the minimum required to perform the critical task.

As a practical problem, it is particularly important to realize that in a simulation, either ground or flight, control power cannot be varied by means of a simple gain changer (such as a potentiometer, etc.) if the control system sensitivity is to remain constant. Either the physical limits of control stick movement must be varied or additional sophistication, such as a limiter, must be employed (although at the risk of complicating any subsequent interpretation of the data). The more widely supported "control power-damping" experiments reported in the VTOL literature do not reflect that such a precaution has been taken. In the opinion of this author, this omission has probably accounted for much of the conflicting control power data that exist today.

Indeed, many of these past experiments have probably obtained POR data that are more directly a function of control sensitivity than of control power by virtue of the experimental techniques. Witness, for example, paragraph two, page 3 of Ref. 19, where it is stated that the maximum available pitch control stick deflection was "beyond the normal range of stick motions." The inclusion of rate damping as an additional parameter in such experiments and, usually, the lack of enough information about the airframe dynamics (simulated or real) or the control tasks rated by the pilot have served to compound the difficulties.

The analytical approach to the specification of VTOL control power may prove to be of greatest value during preliminary airframe design stages when, even with estimated system characteristics, the methods of this paper can permit realistic comparisons of competing systems without the necessity for simulation and with minimum requirements for empirical data.

In conclusion, it should be remarked that the specification of valid control power requirements for operational VTOL aircraft is a complicated process, one that does not appear suitable for the use of empirical methods so often encountered in "practical" engineering design. It is hoped that this paper will, in some way, stimulate the employment of more logical, imaginative thought in the design of VTOL aircraft control systems than has often been exhibited in the past.

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